Desert Dune Dynamics And Processes

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Abstract: The development of the dunes are governed by the effects of turbulence. Turbulence is a type of fluid flow that is strongly rotational and apparently chaotic. Turbulence separates nearby parcels of air and thus mixed fluid properties. The evolution of sand dunes is determined by the interactions between the atmosphere, the surface and the transport and deposition of sand. We are concerned with this physical process and its computational simulation from three perspectives; namely, (1) flow structure; (2) sand transport and deposition and (3) interactions between flow structure and sand transport-deposition, which determine the dune morphology.

Keywords: Coastal management, dunes, sediment supply.

Introduction

The system of moving bedforms in a flow field can be explained by the sediment-continuity equation and the sediment-transport equation. It yields

\[ \frac{\partial h(x,t)}{\partial t} = \frac{1}{s} \frac{\partial q(x,t)}{\partial x} \]  \hspace{1cm} (1)

\[ q(x,t) = f(t \times t) \]  \hspace{1cm} (2)

where \( h \) is the height of the bedforms or topographic height, \( t \) is the time, \( \sigma \) is the sediment density, \( q \) is the sediment transport capacity in kg.m\(^{-1}\).s\(^{-1}\), \( \tau \) is the shear stress due to saltation.

Mathematical Model

Equation (2) shows that the sediment transport varies linearly with the shear stress (Stam, 1994) as

\[ q(x,t) = q_o + A_1 \tau(x,t) \]  \hspace{1cm} (3)

where \( q_o \) is a constant basic sediment-transport and \( A_1 \) is the linearity constant. If topographic variations are relatively small, it can be generally stated that the shear stress is formed of a constant basic term \((\tau_o)\), and a correction term \((\tau_1)\) that varies with space and time:

\[ \tau(x,t) = \tau_o + \tau_1(x,t) \]  \hspace{1cm} (4)

If, in a first approximation, this correction is assumed to be linear with the topographic height:

\[ \tau_1(x,t) = A_2 h(x,t) \]  \hspace{1cm} (5)

the sediment-transport equation becomes:

\[ q(x,t) = q_o + A_1 \tau_o + A_3 h(x,t) \]  \hspace{1cm} (6)

where \( A_3 = A/A_2 \)

Taking the derivative of Equation (6), it yields

\[ \frac{\partial q(x,t)}{\partial x} = A_3 \frac{\partial h(x,t)}{\partial x} \]  \hspace{1cm} (7)

which, substituted in the continuity equation becomes

\[ \frac{\partial h(x,t)}{\partial t} + \frac{A_3}{s} \frac{\partial h(x,t)}{\partial x} = 0 \]  \hspace{1cm} (8)
This is called the simple wave equation, and it describes the propagation of a wave at constant velocity. The ratio

$$c = \frac{A_3}{s} \quad (9)$$

The simple-wave equation, in this case, results from combining a linear shear stress and a linear sediment-transport formula. The ratio in Equation (9) is called the wave velocity. It concerns the morphodynamics of dunes and this wave velocity is equal to the migration rate. This wave will advance at a constant rate $c$ without changing its shape. The solution of Equation (8) is given as

$$h(x,t)=f(x – ct) \quad (10)$$

which expresses that at a certain point $x$ at a certain time $t$ the topography will have the same height as it had at the start (at $t=0$) at a point $(x – ct)$. The height of the bedforms can be any function of $(x – ct)$.

**Kinematic Wave Approximation**

Another formulation of the mathematical model of the topography is given by using a sediment-transport formula instead of a linear relationship with the shear stress. Bagnold’s (1941) model yields

$$q(x,t) = C_B (x,t)^{3/2} \quad (11)$$

where $C_B$ is the Bagnold’s constants with the unit $[s^{2/3}m^{1/3}kg^{1/2}]$. If the linear shear stress are assumed as in the Equations (4) and (5), it is obtained

$$q(x,t) = C_B (t_o + A_2 h(x,t))^{3/2} \quad (12)$$

and

$$\frac{\partial q(x,t)}{\partial x} = \frac{3}{2} \frac{C_B}{s} (t_o + A_2 h(x,t))^{1/2} A_2 \frac{\partial h(x,t)}{\partial x}$$

$$\frac{\partial h(x,t)}{\partial t} = -\frac{3}{2} \frac{C_B}{s} A_2 (t_o + A_2 h(x,t))^{1/2} \frac{\partial h(x,t)}{\partial x} \quad (13) \text{ and } (14)$$

In this mathematical model it can be seen that the migration rate is not a constant but a more complicated expression that varies with the topographic height, and the wave velocity is given as

$$c = \frac{3}{2} \frac{C_B}{s} A_2 (t_o + A_2 h(x,t))^{1/2} \quad (15)$$

In this mathematical model the dune shape changes. The migration rate increases with height, which means that the top of the dune will advance more rapidly than the base. The lee side of the dune will tend to become steeper and the peak will eventually overtake the slipface. In observations, according to this mathematical model, the maximum angle of repose for sediment will be surpassed and avalanching will occur at the slipface, limiting this asymmetrical shape. This is a well known type of equation called a breaking-wave equation which shows the breaking-wave behaviour at the mathematical solution. This behaviour of the solution will be similar for any non-linear sediment-transport equation, as long as the peak has a higher velocity than the base which is given in mathematical formulation as

$$\frac{\partial c(h)}{\partial h} > 0 \quad (16)$$

The deformation of the bedform in a “dune-like shape” is partly due to the non-linear relationship between sediment transport and shear stress.
Development of an analytical solution for bedform migration and growth

The expression of the shear stress of the velocity in terms of the topography makes it adequate for the development of an analytical solution. The analytical solution becomes simpler if only one wave number is considered, so that the summations in the Fourier series are reduced to only one term.

Dimensionless coordinates have been used (indicated by an asterisk *), so that

\[
\frac{\partial h^*(x^*, t)}{\partial t} = \frac{1}{s^*} \frac{\partial q(x^*, t)}{\partial x^*} \\
\]

(17)

where \( H \) is the maximum topographic height [m].

A spatial derivative expression of the sediment transport has to be substituted in the continuity equation. A linearization of Bagnold’s sediment-transport formula will be used (Stam, 1994). For the linearization it has to be considered that the shear stress (\( \tau(x^*, t) \)) results from Prandtl’s logarithmic profile (\( \tau_o \)) with Jackson and Hunt’s (1975) dimensionless first-order correction (\( \tau_1(x^*, t) \))

\[
t(x^*, t) = \tau_o \left( 1 + \varepsilon \tau_1(x^*, t) \right) \\
\]

(21)

Bagnold’s linearized sediment-transport formula then becomes

\[
q_1(x^*, t) = C_B \tau_o^{3/2} \left( 1 + \frac{3}{2} \varepsilon \tau_1(x^*, t) \right) \\
\]

(22)

where:

\( \tau_o \) = shear stress from logarithmic profile [Pa]
\( \tau_1 \) = first order correction to the shear stress from the logarithmic profile [dimensionless]
\( \varepsilon \) = perturbation factor. This is dimensionless number smaller tan 1. (Stam, 1994)

Differentiation of the linearized transport equation to the dimensionless coordinate \( x^* \) gives

\[
\frac{\partial q_1(x^*, t)}{\partial x^*} = \frac{3}{2} C_B \varepsilon \tau_o^{3/2} \frac{\partial \tau_1(x^*, t)}{\partial x^*} \\
\]

(23)
correction of the shear stress can be used for the developing of this equation. Expressed as a Fourier Transform $\tau(k^*,t)$ the shear stress correction is given as

$$
\tau_1(k^*,t) = k^* h(k^*,t) e^{i\phi} \frac{K_1(z_k e^{i\phi})}{K_0(z_k e^{i\phi})}
$$

where:
- $l =$ thickness of the inner region [m]
- $K_0 =$ Modified Bessel function of the zero-order
- $K_1 =$ Modified Bessel function of the first-order

$k^*$= dimensionless wave number. It should be noted that by introducing the dimensionless coordinate $x^* = x/L$, the maximum wave length ($\lambda$) has become equal to one and therefore the wave number ($k=2\pi/\lambda$) has become dimensionless also. For small arguments, the

$$
z_k = 2 \sqrt{z_r \frac{|k^*|}{l}}
$$

and $\phi = \pi/4$ if $k^*>0$ (positive wave number)

$$
\phi = -\pi/4 \text{ if } k^*<0 \text{ (negative wave number)}
$$

Conclusions

The evolution of desert dunes is determined by the interactions between the atmosphere, the surface and the transport and deposition of sand while the morphology and dynamics of Mediterranean Aeolian sand dunes are governed by sand movement induced by shore wave shear. In conditions of unidirectional constant winds and sand supply, it is well known that transverse and Mediterranean shore dunes migrate downwind without changing their shapes in comparison with the desert barchan dunes. Beach ridges or coastal dunes consist of also compound dunes made up of two or more dunes of the same basic type, coalescing or overlapping, and complex dunes in which two or more different basic types are combined or superimposed.

References


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